# A Technical Appendix

*Proof of Proposition 1.* We begin by finding the segment of consumers who would buy the good based only on their priors; that is, those whose expected utility is such that:

$$EU^{BB}(a,p) = \frac{1}{2} + a - p \geq 0 \iff a \geq a_0 \equiv p - \frac{1}{2}.$$

We know that the participation cut-off $a_0$ always falls in the region where the types are supported: $a_0 \in (0,1)\, \forall p \in \left(\frac{1}{2}, 1\right)$.

Thus, consumers with types $a \in (p, 1]$ would be willing to demand the expert's services given the good's price and their type. After consulting the expert, the consumer acquires the good if its quality is high enough. That is:

$$U^{ex-post}(q,a,p) = q + a - p \geq 0 \iff q \geq q^X \equiv p - a.$$

The minimum quality will fall in the supported values for the variable if:

$$q^X \geq 0 \iff a \leq p, \quad \text{and}$$
$$q^X \leq 1 \iff a \geq p - 1.$$

Therefore, the consumers who consult the expert will obtain a positive ex-post utility from consulting the expert and buying the good (*i.e.*, the information will be *useful* to them) if the quality reported is $q \in \left[q^X, 1\right]$ and the consumer's type is $a \in [0,p]\ \forall p \in \left(\frac{1}{2}, 1\right)$. No consumer with a type superior to $p$ will ever consider consulting the expert before purchase, no matter how small $\lambda$ is.

Hence, the expected utility from consulting the expert is given by:

$$EU^{XP}(a,p) = \begin{cases} \int_{p-a}^{1} (q + a - p)dq - \lambda & \text{if } a \in [0,p] \\ 0 & \text{otherwise.} \end{cases}$$

An expression we can rewrite as follows:

$$EU^{XP}(a,p) = \begin{cases} \frac{(1+a-p)^2}{2} - \lambda & : \text{ if } a \in [0,p] \\ 0 & \text{otherwise.} \end{cases}$$

We now consider the participation decision of the consumers who may be willing to consult the expert. For that to be the case, the expected utility obtained must be positive and superior to what the consumers would get from buying the good based on their priors. That is:

$$EU^{XP}(a,p) \geq 0 \iff a \geq a_1 \equiv p - 1 + \sqrt{2\lambda},$$
$$EU^{XP}(a,p) \geq EU^{BB}(a,p) \iff a \leq a_2 \equiv p - \sqrt{2\lambda}.$$

We can easily see that $a_1 > 0 \iff \lambda > \frac{(1-p)^2}{2}$, $a_1 < p \iff \lambda < \frac{1}{2}$ and $a_2 > 0 \iff \lambda < \frac{p^2}{2}$. Therefore, the relevant values for the type are:

$$EU^{XP}(a,p) \geq 0 \text{ for all } a \in [0,1] \text{ if } \lambda \in \left[0, \frac{(1-p)^2}{2}\right] \text{ or}$$
$$\text{for all } a \in [a_1, 1] \text{ if } \lambda \in \left(\frac{(1-p)^2}{2}, \frac{1}{2}\right),$$
$$EU^{XP}(a,p) \geq EU^{BB}(a,p) \text{ for all } a \in [0, a_2] \text{ if } \lambda \in \left[0, \frac{p^2}{2}\right].$$

Also, notice that $a_2 > a_1 \iff \lambda \in \left(0, \frac{1}{8}\right)$.

With this information we can build the demand system for the expert, conditional on the fee he charges and the price of the good.

First, consider the case where $\lambda \in \left(0, \frac{(1-p)^2}{2}\right]$. A graphic representation of the demand faced by the expert, considering the arrangement of the relevant cut-off levels, is given by the dashed segment:


Figure 1: Expert services market when $\lambda \in \left(0, \frac{(1-p)^2}{2}\right]$ and $p \in \left(\frac{1}{2}, 1\right)$

Where $a_1 < 0$ implies that for the given $\lambda$ and $p$, $EU^{XP}(p, \lambda) > 0\ \forall a \in [0, 1]$. Moreover, $EU^{XP}(p, \lambda) \geq EU^{BB}(p, \lambda)\ \forall a \in [0, a_2]$. Hence, the demand for expert services in this case is given by:

$$D^{XP}(\lambda, p) = a_2.$$

Next, consider the case where $\lambda \in \left(\frac{(1-p)^2}{2}, \frac{1}{8}\right]$. Again, the demand faced by the expert is given by the dashed segment:


Figure 2: Expert services market when $\lambda \in \left(\frac{(1-p)^2}{2}, \frac{1}{8}\right]$ and $p \in \left(\frac{1}{2}, 1\right)$

Here $a_1 > 0$, which implies that for the given $\lambda$ and $p$, $EU^{XP}(p, \lambda) > 0\ \forall a \in [a_1, 1]$. Moreover, $EU^{XP}(p, \lambda) \geq EU^{BB}(p, \lambda)\ \forall a \in [0, a_2]$. Hence, the demand for expert services in this case is given by:

$$D^{XP}(\lambda, p) = a_2 - a_1.$$

Last, consider the case where $\lambda \in \left(\frac{1}{8}, \frac{p^2}{2}\right]$. Here $a_1 > a_2$, which implies that there is no demand for the expert for the given $\lambda$ and $p$. Hence:

$$D^{XP}(\lambda, p) = 0.$$

Therefore, the demand for expert services can be written as follows:

$$D^{XP}(\lambda, p) = \begin{cases} p - \sqrt{2\lambda} & : \text{ if } \lambda \in \left[0, \frac{(1-p)^2}{2}\right] \\ 1 - 2\sqrt{2\lambda} & : \text{ if } \lambda \in \left[\frac{(1-p)^2}{2}, \frac{1}{8}\right] \\ 0 & \qquad \text{otherwise.} \end{cases}$$

There are two cases to consider, corresponding to each segment of the demand function, when solving the expert's maximization problem. We denote these *Case I* and *II*, such that:

$$
\begin{aligned}
\max_{\lambda} \quad & \Pi^{XP-I} = \lambda(p - \sqrt{2\lambda}) \\
\text{s.t.} \quad & \lambda \geq 0 \\
& \lambda \leq \frac{(1-p)^2}{2},
\end{aligned}
$$

is the maximization problem for *Case I*, and

$$
\begin{aligned}
\max_{\lambda} \quad & \Pi^{XP-II} = \lambda(1 - 2\sqrt{2\lambda}) \\
\text{s.t.} \quad & \lambda \geq \frac{(1-p)^2}{2} \\
& \lambda \leq \frac{1}{8},
\end{aligned}
$$

is the maximization problem for *Case II*.

From the respective Kuhn-Tucker conditions we find that each maximization problem has two valid solutions, depending on the size of $p$. For *Case I*:

$$
\lambda_1^I = \frac{2}{9}p^2 \ \text{ if } p \in \left(\frac{1}{2}, \frac{3}{5}\right] \quad \text{and } \lambda_2^I = \frac{(1-p)^2}{2} \ \text{ if } p \in \left(\frac{3}{5}, 1\right).
$$

And for *Case II*:

$$
\lambda_1^{II} = \frac{(1-p)^2}{2} \ \text{ if } p \in \left(\frac{1}{2}, \frac{2}{3}\right] \quad \text{and } \lambda_2^{II} = \frac{1}{18} \ \text{ if } p \in \left(\frac{2}{3}, 1\right).
$$

However, one can easily find the expert's optimal fee for each pricing region. When $p \in \left(\frac{1}{2}, \frac{3}{5}\right]$ both $\lambda_1^I$ and $\lambda_1^{II}$ are feasible candidates, but $\lambda_1^I$ dominates the other since they are respectively an internal and corner solution for the maximization problem under the established values of $p$. The same happens when $p \in \left(\frac{2}{3}, 1\right)$, where both $\lambda_2^I$ and $\lambda_2^{II}$ are valid but the latter dominates the former, being an interior solution. There is only one valid candidate when $p \in \left(\frac{3}{5}, \frac{2}{3}\right]$: $\lambda_2^I = \lambda_1^{II} = \frac{(1-p)^2}{2}$.

The demand the expert serves and the profits he obtains given a pricing level, are:

$$
\begin{aligned}
&\text{If } \ p \in \left(\frac{1}{2}, \frac{3}{5}\right] && \text{then } \ D^{XP} = \frac{p}{3}, \ \Pi^{XP} = \frac{2}{27}p^3. \\
&\text{If } \ p \in \left(\frac{3}{5}, \frac{2}{3}\right] && \text{then } \ D^{XP} = 2p - 1, \ \Pi^{XP} = \frac{(1-p)^2}{2}(2p-1). \\
&\text{If } \ p \in \left(\frac{2}{3}, 1\right) && \text{then } \ D^{XP} = \frac{1}{3}, \ \Pi^{XP} = \frac{1}{54}.
\end{aligned}
$$

□

*Proof of Lemma 1.* The segment of consumers who would be willing to buy the good based only on their expectations is:

$$
EU^{BB}(a, p) = \frac{1}{2} + a - p \geq 0 \iff a \geq a_0 \equiv p - \frac{1}{2}.
$$

Clearly, for the values of $p$ that the firm can set, the participation cut-off computed falls in the region where the types are supported: $a_0 \in (0,1)\, \forall p \in \left(\frac{1}{2}, 1\right)$.

Therefore, the demand is given by:

$$D^G = 1 - a_0 = \frac{3}{2} - p.$$

The firm's maximization problem is the following:

$$\max_p \quad \Pi^G = p\left(\frac{3}{2} - p\right).$$

From which we find that the optimal price is:

$$p = \frac{3}{4}.$$

The demand the firm serves is $D^G = \frac{3}{4}$, obtaining profits $\Pi^G = \left(\frac{3}{4}\right)^2$. $\square$

*Proof of Proposition 2.* From the proof of *Proposition 1* we know that consumers with a type $a \in [a_0, 1]$ obtain a positive utility from buying the good based only on their priors.

We also know the expected utility for those consumers who buy the good after consulting the expert:

$$EU^{XP}(a,p) = \begin{cases} \frac{(1+a-p)^2}{2} - \lambda & : \text{ if } a \in [0,p] \\ 0 & \quad \text{otherwise.} \end{cases}$$

Moreover:

$$EU^{XP}(a,p) \geq 0 \iff a \geq a_1 \equiv p - 1 + \sqrt{2\lambda},$$
$$EU^{XP}(a,p) \geq EU^{BB}(a,p) \iff a \leq a_2 \equiv p - \sqrt{2\lambda}.$$

There are three cases to consider:

1. When the price is in the *low* region, $p \in \left(\frac{1}{2}, \frac{3}{5}\right]$.

   We know that the demand for the good will comprise those consumers who would have bought the good based only on their priors and those who, once they learn $q$ from the expert, obtain a positive *ex post* utility. That is, those consumers who ask the expert and learn that the quality is at least $q^X \equiv p - a$. The expert sets an optimal fee $\lambda = \frac{2p^2}{9}$ in this region. Therefore, the demand for the good is given by:

   $$D^G = (1 - a_2) + \int_0^{a_2} (1 - (p - a))da = 1 - \frac{5p^2}{18}.$$

   From solving the maximization problem we get $p = \sqrt{\frac{6}{5}}$ as a candidate solution. However, it falls outside of the supported pricing region, being bigger than 1. Hence, the maximization problem's solution is not interior, taking the maximum value for the price: $p^G = \frac{3}{5}$, with profits $\Pi^G = \frac{27}{50}$.

2. When the price is in the *intermediate* region, $p \in \left(\frac{3}{5}, \frac{2}{3}\right]$.

   Here the demand continues to be given by:

$$D^G = (1 - a_2) + \int_0^{a_2} (1 - (p - a))da,$$

   although the fee charged by the expert is: $\lambda = \frac{(1-p)^2}{2}$. Therefore, the demand for the good in this case is given by:

$$D^G = \frac{3}{2} - p.$$

   From solving the maximization problem we get the candidate solution $p = \frac{3}{4}$. However, it falls outside of the supporting pricing region. Thus, the optimal price set by the firm is: $p^G = \frac{2}{3}$, obtaining profits for $\Pi^G = \frac{5}{9}$.

3. Finally, when the price is in the *high* region, $p \in \left(\frac{2}{3}, 1\right]$.

   As in the previous two cases, the demand for the good is given by:

$$D^G = (1 - a_2) + \int_0^{a_2} (1 - (p - a))da,$$

   with the expert charging a fee $\lambda = \frac{1}{18}$. Therefore, the demand for the good in this pricing region is:

$$D^G = \frac{3}{2} - p.$$

   In this case the candidate solution obtained from solving the maximization problem $p = \frac{3}{4}$ is supported by the pricing region. Hence, the optimal price set by the firm is $p^G = \frac{3}{4}$, with profits $\Pi^G = \frac{9}{16}$.

By comparing the different profits levels we can see that the expert gets the highest profits when setting a price in the *high* region. Thus, his optimal fee is $p^G = \frac{3}{4}$.

□

*Proof of Proposition 3.* This proof follows the general structure of *Proposition 1*'s proof, although we adjust the consumers' decisions to include the new information available from user reviews (from now on UR). Therefore, we must consider two cases: *Case A* when the UR tell the consumers that the good's quality is above $\frac{1}{2}$, and *Case B* when the UR reveal the quality of the good to be below $\frac{1}{2}$.

We begin by studying *Case A*, where $q \geq \frac{1}{2}$. Upon seeing a *star review* from the UR, the consumers update their priors on the good's quality, such that: $q \sim U\left(\frac{1}{2}, 1\right)$. Thus, the consumers' expected value for the quality is $\int_{\frac{1}{2}}^{1} 2q\, dq = \frac{3}{4}$. Hence, the expected utility for the consumers who purchase without consulting the expert is:

$$EU^{BB}(p, a) = \frac{3}{4} + a - p.$$

Furthermore, consumers with a type such that:

$$EU^{BB}(p,a) \geq 0 \iff a \geq a_{BB} \equiv p - \frac{3}{4},$$

will consider buying the good based solely on their UR-updated priors. Notice that $a_{BB} \geq 0 \iff p \geq \frac{3}{4}$. Thus, we need to consider two participation scenarios: the first when $p \in \left(\frac{1}{2}, \frac{3}{4}\right]$ so that any consumer in the market obtains positive expected utility from buying the good based on the UR, and the second when $p \in \left(\frac{3}{4}, 1\right)$ and only consumers with $a \in (a_{BB}, 1]$ would buy the good based on the information coming from the UR.

A consumer who reads the UR and potentially considers consulting the expert before buying, cannot have a type parameter such that his utility from purchasing based on his UR-updated priors is positive even when the quality of the good takes the lowest value ($q = \frac{1}{2}$). That is:

$$EU^{min}(a,p) = \frac{1}{2} + a - p \geq 0 \iff a \geq a_0 \equiv p - \frac{1}{2}.$$

Thus, the segment of consumers who may consult the expert have a type in the following region: $a \in [0, a_0]$. Notice that $a_{BB} < a_0$ for any value of $p$. Then, there may be a potential demand for the expert between consumers who observe the star review and still want to consult with him before buying the good. These are the consumers with types in the $a \in (a_{BB}, a_0]$ segment.

Out of those consumers some will be interested in asking the expert, given the good's price and their own type, if the quality revealed is high enough for them to obtain an *ex post* positive utility. That is:

$$U^{ex-post}(q,a,p) = q + a - p \geq 0 \iff q \geq q^X \equiv p - a.$$

The minimum quality will fall in the supported interval if:

$$\begin{aligned} q^X \geq \frac{1}{2} &\iff a \leq a_0 \equiv p - \frac{1}{2}, \quad \text{and} \\ q^X \leq 1 &\iff a \geq a_1 \equiv p - 1. \end{aligned}$$

Notice that $a_1 < 0$ for $p \in \left(\frac{1}{2}, 1\right)$. Therefore, the expected utility consumers with types $a \in [0, a_0]$ obtain from consulting the expert is given by:

$$EU^{XP}(a,p) = \begin{cases} 2\int_{p-a}^{1}(q+a-p)dq - \lambda = (1+a-p)^2 - \lambda & : \text{ if } a \in [0, a_0] \\ 0 & \text{otherwise.} \end{cases}$$

We now consider the participation decisions of the consumers who may be willing to consult the expert. The expected utility they obtain must be positive and superior to what the consumers would get from buying the good based on their priors. That is:

$$\begin{aligned} &EU^{XP}(a,p) \geq 0 \iff a \leq a_2 \equiv p - 1 - \sqrt{\lambda} \text{ or } a \geq a_3 \equiv p - 1 + \sqrt{\lambda} \\ &\qquad\qquad \text{and} \\ &EU^{XP}(a,p) \geq EU^{BB}(a,p) \iff a \leq a_4 \equiv p - \frac{1}{2} - \sqrt{\lambda} \text{ or } a \geq a_5 \equiv p - \frac{1}{2} + \sqrt{\lambda}. \end{aligned}$$

However, not all of the cut-offs computed fall in the supported region for the types. We can easily see that $a_2 < 0$ and $a_5 > a_0$ for $p \in \left(\frac{1}{2}, 1\right)$ and $\lambda > 0$. We thus discard these.

Furthermore:

$$a_3 > 0 \iff \lambda > (1-p)^2 \text{ and}$$
$$a_3 < a_0 \iff \lambda < \frac{1}{4} \text{ and}$$
$$a_3 < a_{BB} \iff \lambda < \frac{1}{16}.$$

Similarly:

$$a_4 > 0 \iff \lambda < \frac{(1-2p)^2}{4} \text{ and}$$
$$a_4 < a_0 \text{ for } \lambda > 0 \text{ and } p \in \left(\frac{1}{2}, 1\right), \text{ and}$$
$$a_4 < a_{BB} \iff \lambda > \frac{1}{16}.$$

Also, notice that:

$$a_4 > a_3 \iff \lambda \in \left(0, \frac{1}{16}\right).$$

Hence, we know that the expert faces no demand whenever he charges a fee higher than $\frac{1}{16}$.

We first study *Case I*, where $p \in \left(\frac{1}{2}, \frac{3}{4}\right]$. We know that for this pricing level $(1-p)^2 > \frac{1}{16} > \frac{(1-2p)^2}{4} > 0$. With this information we can build the demand system for the expert, conditional on the fee he charges and the good's price.

First, consider the case where $\lambda \in \left(0, \frac{(1-2p)^2}{4}\right]$. A graphic representation of the demand faced by the expert, considering the arrangement of the relevant cut-off levels, is given by the dashed segment:

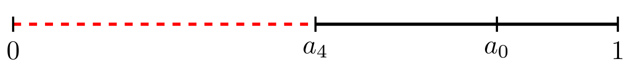


Figure 3: Expert services market when $\lambda \in \left(0, \frac{(1-2p)^2}{4}\right]$ and $p \in \left(\frac{1}{2}, \frac{3}{4}\right]$

Where $a_3 < 0$ implies that for the given $\lambda$ and $p$, $EU^{XP}(p, \lambda) > 0 \ \forall a \in [0, a_0]$. Moreover, $EU^{XP}(p, \lambda) \geq EU^{BB}(p, \lambda) \ \forall a \in [0, a_4]$. Hence, the demand for expert services in this case is given by:

$$D^{XP-A}(\lambda, p) = a_4 = p - \frac{1}{2} - \sqrt{\lambda}.$$

Next, we consider the case where $\lambda \in \left(\frac{(1-2p)^2}{4}, \frac{1}{16}\right]$. Here the size of $\lambda$ implies that $a_4 < 0$, which means that no consumer can get a higher utility from consulting the expert than when buying the good based on their UR-updated priors; the fee is just too high to compensate the value of the information obtained from the expert. Hence, there is no demand for the expert for the given values of $\lambda$ and $p$:

$$D^{XP-A}(\lambda, p) = 0.$$

Now we move to *Case II*, where $p \in \left(\frac{3}{4}, 1\right)$. Thus, we know that for this pricing level: $\frac{(1-2p)^2}{4} > \frac{1}{16} > (1-p)^2 > 0$.

First, consider the case where $\lambda \in \left(0, (1-p)^2\right]$. A graphic representation of the demand faced by the expert, considering the arrangement of the relevant cut-off levels, is given by the dashed segment:

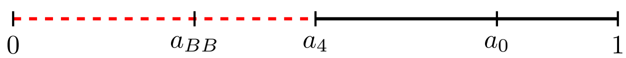


Figure 4: Expert services market when $\lambda \in \left(0, (1-p)^2\right]$ and $p \in \left(\frac{3}{4}, 1\right)$

Where $a_3 < 0$ implies that for the given $\lambda$ and $p$, $EU^{XP}(p, \lambda) > 0$ $\forall a \in [0, a_0]$. Moreover, $EU^{XP}(p, \lambda) \geq EU^{BB}(p, \lambda)$ $\forall a \in [0, a_4]$. Hence, the demand for expert services in this case is given by:

$$D^{XP-A}(\lambda, p) = a_4 = p - \frac{1}{2} - \sqrt{\lambda}.$$

Next, consider the case where $\lambda \in \left((1-p)^2, \frac{1}{16}\right]$. Again, the demand faced by the expert is given by the dashed segment:


Figure 5: Expert services market when $\lambda \in \left((1-p)^2, \frac{1}{16}\right]$ and $p \in \left(\frac{3}{4}, 1\right)$

Here $a_3 > 0$ implies that for the given $\lambda$ and $p$, $EU^{XP}(p, \lambda) > 0$ $\forall a \in [a_3, a_0]$. Moreover, $EU^{XP}(p, \lambda) \geq EU^{BB}(p, \lambda)$ $\forall a \in [0, a_4]$. Hence, the demand for expert services in this case is given by:

$$D^{XP-A}(\lambda, p) = a_4 - a_3 = \frac{1}{2} - 2\sqrt{\lambda}.$$

Lastly, consider the case where $\lambda > \frac{1}{16}$. Here $a_3 > a_4$, which implies that there is no demand for the expert for the given levels of $\lambda$ and $p$. Hence:

$$D^{XP-A}(\lambda, p) = 0.$$

Having completed the analysis of the case where the good's quality is revealed by the UR to be above the expected value, we move to *Case B*, where $q < \frac{1}{2}$. That is, the consumers do not see a *star review* from the users, updating their priors on the good's quality such that: $q \sim U\left(0, \frac{1}{2}\right)$. Thus, the consumers' expected value for the quality is $\frac{1}{4}$. Hence, the expected utility for the consumers who purchase without consulting the expert, is:

$$EU^{BB-2}(p, a) = \frac{1}{4} + a - p.$$

Furthermore, consumers with a type such that:

$$EU^{BB-2}(p, a) \geq 0 \iff a \geq a_{BB-2} \equiv p - \frac{1}{4},$$

will consider buying the good based only on their UR-updated priors. Notice that $a_{BB-2} \geq 0 \iff p \geq \frac{1}{4}$, which is always the case for $p \in \left(\frac{1}{2}, 1\right)$. Thus, consumers with $a \in (a_{BB-2}, 1)$ would buy the good based on the information coming from the UR.

A consumer who reads the UR and potentially considers consulting the expert before buying cannot have a type parameter such that his utility from purchasing based on his UR-updated priors is positive even when the quality takes the lowest value possible ($q = 0$). That is:

$$EU^{min}(a,p) = 0 + a - p \geq 0 \iff a \geq a_{0-B} \equiv p.$$

Thus, the segment of consumers who may consult the expert have a type in the region $[0, a_{0-B})$. Notice that $a_{0-B} > a_{BB-2}$ for $p \in \left(\frac{1}{2}, 1\right)$.

Out of these consumers some will be interested in consulting the expert, given the good's price and their own type, if the quality revealed is high enough for them to obtain an *ex post* positive utility. That is:

$$U^{ex-post}(q,a,p) = q + a - p \geq 0 \iff q \geq q^{X-B} \equiv p - a.$$

The minimum quality will fall in the supported values for the variable if:

$$q^{X-B} \geq 0 \iff a \leq a_{0-B} \equiv p, \quad \text{and}$$
$$q^{X-B} \leq \frac{1}{2} \iff a \geq a_{1-B} \equiv p - \frac{1}{2}.$$

Where $a_{1-B} \in (0, a_{BB-2})$ for $p \in \left(\frac{1}{2}, 1\right)$. Thus, the consumers potentially ask the expert if and only if their type is $a \in [a_{1-B}, a_{0-B}]$. Consumers with higher or lower type values either buy the good based on their own priors or just stay out of the market.

The consumers' expected utility from consulting the expert is given by:

$$EU^{XP}(a,p) = \begin{cases} 2\int_{p-a}^{\frac{1}{2}}(q + a - p)dq - \lambda = \frac{1}{4}(1 + 2a - 2p)^2 - \lambda & : \text{ if } a \in [a_{1-B}, a_{0-B}] \\ 0 & \text{otherwise.} \end{cases}$$

We now consider the participation decisions of the consumers who may be willing to consult the expert. We proceed as in this proof's first case:

$$EU^{XP}(a,p) \geq 0 \iff a \geq a_{2-B1} \equiv p - \frac{1}{2} + \sqrt{\lambda} \text{ or },$$
$$a \leq a_{2-B2} \equiv p - \frac{1}{2} - \sqrt{\lambda}.$$

and

$$EU^{XP}(a,p) \geq EU^{BB-2}(a,p) \iff a \leq a_{3-B} \equiv p - \sqrt{\lambda} \text{ or},$$
$$a \geq a_{4-B} \equiv p + \sqrt{\lambda}.$$

However, not all of the cut-offs computed fall in the supported region. We can easily see that $a_{1-B} > a_{2-B2}$ and $a_{4-B} > a_{0-B}$ for all values of $p$ and $\lambda$. We thus discard $a_{2-B2}$ and $a_{4-B}$. Furthermore:

$$a_{2-B1} > a_{1-B} \text{ for } p \in \left(\frac{1}{2}, 1\right) \text{ and } \lambda > 0, \quad \text{and}$$
$$a_{2-B1} > a_{0-B} \iff \lambda > \frac{1}{4}, \quad \text{and}$$
$$a_{2-B1} < a_{BB-2} \iff \lambda \leq \frac{1}{16}.$$

Similarly

$$
\begin{aligned}
a_{3-B} &< a_{0-B} \;\text{ for }\; p \in \left(\frac{1}{2}, 1\right) \;\text{ and } \lambda > 0, \quad \text{ and} \\
a_{3-B} &> a_{1-B} \iff \lambda \leq \frac{1}{4}, \quad \text{ and} \\
a_{3-B} &< a_{BB-2} \iff \lambda > \frac{1}{16}.
\end{aligned}
$$

Also notice that:

$$
\begin{aligned}
a_{3-B} \geq a_{BB-2} \geq a_{2B-1} &\iff \lambda \in \left(0, \frac{1}{16}\right] \;\text{ and} \\
a_{2B-1} > a_{BB-2} > a_{3-B} &\iff \lambda > \frac{1}{16}
\end{aligned}
$$

We need to consider two cases when computing the demand faced by the expert: *Case I - B* when $\lambda \in \left(0, \frac{1}{16}\right]$ and *Case II - B* when $\lambda \in \left(\frac{1}{16}, \frac{1}{4}\right]$. The expert faces no demand when charging higher fees.

We begin the analysis of the demand with *Case I-B.* A graphic representation of the demand faced by the expert, considering the arrangement of the relevant cut-off levels, is given by the dashed segment:


Figure 6: Expert services market when $\lambda \in \left(0, \frac{1}{16}\right]$ and $p \in \left(\frac{1}{2}, 1\right]$

Where for the given values of $\lambda$ and $p$, $EU^{XP}(p,\lambda) > 0 \;\forall a \;\in [a_{2B-1}, a_{0-B}]$. Moreover, $EU^{XP}(p,\lambda) \geq EU^{BB-2}(p,\lambda) \;\forall a \;\in [a_{1-B}, a_{3-B}]$. Hence, the demand for expert services in this case is given by:

$$
D^{XP-B}(\lambda, p) = a_{3-B} - a_{2B-1} = \frac{1}{2} - 2\sqrt{\lambda}.
$$

Next, consider the case where $\lambda \in \left(\frac{1}{16}, \frac{1}{4}\right]$. Charging a fee on this level implies that $a_{2B-1} > a_{3-B}$; hence, no consumer obtains a positive expected utility from buying the good after consulting the expert. Therefore, the expert faces no demand when charging a fee in this level:

$$
D^{XP}(\lambda, p) = 0.
$$

We can now write the demand for expert services, corresponding to each of the good's pricing levels. In each case, the demand comprises the expected sum of what the expert would face when the good's quality is above and below $\frac{1}{2}$, respectively: $ED^{XP} = \frac{1}{2}D^{XP-A} + \frac{1}{2}D^{XP-B}$.

For $p \in \left(\frac{1}{2}, \frac{3}{4}\right]$, the expected demand is given by:

$$
ED^{XP-I}(\lambda, p) = \begin{cases}
\frac{1}{2}(p - \frac{1}{2} - \sqrt{\lambda}) + \frac{1}{2}\left(\frac{1}{2} - 2\sqrt{\lambda}\right) = \frac{1}{2}\left(p - 3\sqrt{\lambda}\right) & : \text{ if } \lambda \in \left[0, \frac{(1-2p)^2}{4}\right] \\
\frac{1}{2}(0) + \frac{1}{2}\left(\frac{1}{2} - 2\sqrt{\lambda}\right) = \frac{1}{4} - \sqrt{\lambda} & : \text{ if } \lambda \in \left(\frac{(1-2p)^2}{4}, \frac{1}{16}\right] \\
0 & \text{otherwise.}
\end{cases}
$$

For $p \in \left(\frac{3}{4}, 1\right]$, the expected demand is given by:

$$ED^{XP-II}(\lambda, p) = \begin{cases} \frac{1}{2}\left(p - \frac{1}{2} - \sqrt{\lambda}\right) + \frac{1}{2}\left(\frac{1}{2} - 2\sqrt{\lambda}\right) = \frac{1}{2}\left(p - 3\sqrt{\lambda}\right) & : \text{ if } \lambda \in \left[0, (1-p)^2\right] \\ \frac{1}{2}\left(\frac{1}{2} - 2\sqrt{\lambda}\right) + \frac{1}{2}\left(\frac{1}{2} - 2\sqrt{\lambda}\right) = \frac{1}{2} - 2\sqrt{\lambda} & : \text{ if } \lambda \in \left((1-p)^2, \frac{1}{16}\right] \\ 0 & \text{otherwise.} \end{cases}$$

Since the expert is perfectly informed, he maximizes his profits as he is aware of the demand system just described. We first look at *Case I*, when $p \in \left(\frac{1}{2}, \frac{3}{4}\right]$.

There are two subcases to consider here. The corresponding maximization problems are the following. For *Case I-1*:

$$\begin{aligned} \max_{\lambda} \quad & \Pi^{XP\ I-1} = \lambda\left(\frac{1}{2}\left(p - 3\sqrt{\lambda}\right)\right) \\ \text{s.t.} \quad & \lambda \geq 0 \\ & \lambda \leq \frac{(1-2p)^2}{4} \end{aligned}$$

From the respective Kuhn-Tucker conditions we find that the maximization problem has the following candidate solutions:

$$\lambda_{I-1} = \frac{(1-2p)^2}{4} \text{ if } p \in \left(\frac{1}{2}, \frac{9}{14}\right] \text{ and } \lambda_{I-1B} = \frac{4p^2}{81} \text{ if } p \in \left(\frac{9}{14}, \frac{3}{4}\right],$$

The profit levels associated to each optimal fee, are:

$$\Pi^{XP\ I-1} = \frac{1}{16}(3 - 16p + 28p^2 - 16p^3) \text{ if } p \in \left(\frac{1}{2}, \frac{9}{14}\right] \text{ and } \Pi^{XP\ I-1B} = \frac{2p^3}{243} \text{ if } p \in \left(\frac{9}{14}, \frac{3}{4}\right],$$

The maximization problem for *Case I-2* is:

$$\begin{aligned} \max_{\lambda} \quad & \Pi^{XP\ I-2} = \lambda\left(\frac{1}{4} - \sqrt{\lambda}\right) \\ \text{s.t.} \quad & \lambda \geq \frac{(1-2p)^2}{4} \\ & \lambda \leq \frac{1}{16} \end{aligned}$$

From the respective Kuhn-Tucker conditions we find that the maximization problem has the following candidate solutions:

$$\lambda_{I-2} = \frac{1}{36} \text{ if } p \in \left(\frac{1}{2}, \frac{2}{3}\right] \text{ and } \lambda_{I-2B} = \frac{(1-2p)^2}{4} \text{ if } p \in \left[\frac{2}{3}, \frac{3}{4}\right],$$

The profit levels associated to each optimal fee, are:

$$\Pi^{XP\,I-2} = \frac{1}{432} \text{ if } p \in \left(\frac{1}{2}, \frac{2}{3}\right] \text{ and } \Pi^{XP\,I-2B} = \frac{1}{16}(3 - 16p + 28p^2 - 16p^3) \text{ if } p \in \left[\frac{2}{3}, \frac{3}{4}\right].$$

Finally, from comparing the candidate solutions for *Case I*'s maximization problem we get:

$$\Pi^{XP\,I-1B} > \Pi^{XP\,I-2} \text{ for } p \in \left[\left(\frac{243}{864}\right)^{\frac{1}{3}}, \frac{3}{4}\right].$$

Therefore, depending on the good's pricing level, the expert optimally sets the fees:

$$\lambda = \frac{1}{36} \text{ if } p \in \left(\frac{1}{2}, 0.655\right], \text{ and}$$

$$\lambda = \frac{4p^2}{81} \text{ if } p \in \left(0.655, \frac{3}{4}\right].$$

We now look at *Case II*, when $p \in \left(\frac{3}{4}, 1\right)$.

There are two subcases to consider here. The corresponding maximization problems are the following. For *Case II-1*:

$$\begin{aligned} \max_{\lambda} \quad & \Pi^{XP\,II-1} = \lambda\left(\frac{1}{2}\left(p - 3\sqrt{\lambda}\right)\right) \\ \text{s.t.} \quad & \lambda \geq 0 \\ & \lambda \leq (1-p)^2 \end{aligned}$$

From the respective Kuhn-Tucker conditions we find that the maximization problem has the following candidate solutions:

$$\lambda_{II-1} = \frac{4p^2}{81} \text{ if } p \in \left(\frac{3}{4}, \frac{9}{11}\right] \text{ and } \lambda_{II-1B} = (1-p)^2 \text{ if } p \in \left(\frac{9}{11}, 1\right),$$

The profit levels associated to each optimal fee, are:

$$\Pi^{XP\,II-1} = \frac{2p^3}{243} \text{ if } p \in \left(\frac{3}{4}, \frac{9}{11}\right] \text{ and } \Pi^{XP\,II-1B} = \frac{1}{2}(-3 + 10p - 11p^2 + 4p^3) \text{ if } p \in \left[\frac{9}{11}, 1\right).$$

The maximization problem for *Case II-2* is:

$$\begin{aligned} \max_{\lambda} \quad & \Pi^{XP\,II-2} = \lambda\left(\frac{1}{2} - 2\sqrt{\lambda}\right) \\ \text{s.t.} \quad & \lambda \geq (1-p)^2 \\ & \lambda \leq \frac{1}{16} \end{aligned}$$

From the respective Kuhn-Tucker conditions we find that the maximization problem has the following candidate solutions:

$$\lambda_{II-2} = \frac{1}{36} \text{ if } p \in \left(\frac{5}{6}, 1\right] \text{ and } \lambda_{I-2B} = (1-p)^2 \text{ if } p \in \left(\frac{3}{4}, \frac{5}{6}\right].$$

The profit levels associated to each optimal fee, are:

$$\Pi^{XP\,II-2} = \frac{1}{216} \text{ if } p \in \left(\frac{5}{6}, 1\right] \text{ and } \Pi^{XP\,II-2B} = \frac{1}{2}(-3 + 10p - 11p^2 + 4p^3) \text{ if } p \in \left(\frac{3}{4}, \frac{5}{6}\right].$$

Finally, from comparing the two candidate solutions for *Case II*'s maximization problem we get that depending on the good's pricing level, the expert optimally sets the fees:

$$\lambda = \frac{4p^2}{81} \text{ if } p \in \left(\frac{3}{4}, \frac{9}{11}\right], \text{ and}$$
$$\lambda = (1-p)^2 \text{ if } p \in \left(\frac{9}{11}, \frac{5}{6}\right], \text{ and}$$
$$\lambda = \frac{1}{36} \text{ if } p \in \left(\frac{5}{6}, 1\right).$$

Therefore, the optimal pricing scheme for the expert is:

$$\lambda_* = \begin{cases} \frac{1}{36} & : \text{ if } p \in \left(\frac{1}{2}, 0.6555\right] \\ \frac{4p^2}{81} & : \text{ if } \lambda \in \left(0.6555, \frac{9}{11}\right] \\ (1-p)^2 & : \text{ if } \lambda \in \left(\frac{9}{11}, \frac{5}{6}\right] \\ \frac{1}{36} & : \text{ if } \lambda \in \left(\frac{5}{6}, 1\right) \end{cases}$$

□

*Proof of Proposition 4.* Since the user reviews can take two opposite values, we must consider the firm's decisions in two different cases:

1. When the review is positive: $q \geq \frac{1}{2}$

   In this case the consumer's expected value for the good's quality is $\frac{3}{4}$. Thus, the expected utility for a consumer with type $a$ is given by:

   $$EU^{UR} = \frac{3}{4} + a - p.$$

   When only user reviews and no other sources of information are available in the market, consumers who obtain a positive expected utility decide to buy the good. That is:

   $$EU^{UR} \geq 0 \iff a \geq a_{UR} \equiv p - \frac{3}{4}.$$

   We can see that $a_{UR}$ falls in the support for the type distribution for $p \in \left[\frac{3}{4}, 1\right]$. Therefore, there are two subcases to consider:

- When $p \in \left(\frac{1}{2}, \frac{3}{4}\right]$:
  For these values of $p$, $a_{UR} < 0$. Thus, all consumers with types $a \in [0,1]$ buy the good. The demand for the good is given by:

$$D^{G-S} = 1.$$

- When $p \in \left(\frac{3}{4}, 1\right]$: For these values of $p$, $a_{UR} > 0$. Thus, consumers with types $a \in [a_{UR}, 1]$ buy the good. The demand for the good is given by:

$$D^{G-S1} = 1 - a_{UR} = \frac{7}{4} - p.$$

2. When the review is negative: $q < \frac{1}{2}$
   In this case the consumer's expected value for $q$ is $\frac{1}{4}$. Thus, the expected utility for a consumer with type $a$ is given by:

$$EU^{UR} = \frac{1}{4} + a - p.$$

A consumer will buy the good if:

$$EU^{UR} \geq 0 \iff a \geq a_{UR-2} \equiv p - \frac{1}{4}.$$

We can see that $a_{UR-2}$ falls in the support of the type distribution for any value of $p \in \left(\frac{1}{2}, 1\right)$. Thus, the demand for the good is given by:

$$D^{G-NS} = 1 - a_{UR-2} = \frac{5}{4} - p.$$

We can see that there are two cases to consider when computing the expected demand for the good:

1. If $p \in \left(\frac{1}{2}, \frac{3}{4}\right]$, the expected demand is given by:

$$ED^G = \frac{1}{2}D^{G-S} + \frac{1}{2}D^{G-NS} = \frac{9-4p}{8}.$$

2. If $p \in \left(\frac{3}{4}, 1\right)$, the expected demand is given by:

$$ED^G = \frac{1}{2}D^{G-S1} + \frac{1}{2}D^{G-NS} = \frac{3-2p}{2}.$$

From solving the maximization problem in *Case 1* we find the candidate solution $p = \frac{9}{8}$, which falls outside of the support for the prices. Therefore, we have a corner solution in $p = \frac{3}{4}$. Looking at *Case 2* we find that the candidate solution is also $p = \frac{3}{4}$. Therefore, in the equilibrium the firm charges an optimal price $p^G = \frac{3}{4}$, serves a demand $D^G = \frac{3}{4}$, and obtains profits $\Pi^G = 0.5625$. □

*Proof of Propositions 10.* To find the optimal pricing allocation for the firm we compare the optimal price for each of the pricing regions we have defined. Throughout this proof we use the utility expressions derived in the proof of propositions 4, 5, 6, and 7.

- **When** $p \in \left(\frac{1}{2}, 0.6555\right]$

  In this region the expert charges a fee $\lambda = \frac{1}{16}$. There are two subcases to consider in the *low* pricing region, depending on the size of $q$.

  1. When $q \geq \frac{1}{2}$:
     The relevant consumer decisions to calculate the demand for the good are:

     $$\begin{aligned} EU^{UR} \geq 0 &\iff a \geq a_0 \equiv p - \frac{3}{4}, \\ EU^{XP} \geq 0 &\iff a \geq a_3 \equiv p - 1 + \sqrt{\lambda}, \text{ and,} \\ EU^{XP} \geq EU^{UR} &\iff a \leq a_4 \equiv p - \frac{1}{2} - \sqrt{\lambda}. \end{aligned}$$

     However, $a_0$, $a_3$ and $a_4$ all are smaller than zero for the values of $p$ and $\lambda$ in the region. Thus, the demand for the good when $q \geq \frac{1}{2}$ is given by:

     $$ED^G = 1.$$

  2. When $q < \frac{1}{2}$:
     The relevant consumer decisions to calculate the demand for the good are:

     $$\begin{aligned} EU^{UR} \geq 0 &\iff a \geq a_{0-B} \equiv p - \frac{1}{4}, \\ EU^{XP} \geq 0 &\iff a \geq a_{2B-1} \equiv p - \frac{1}{2} + \sqrt{\lambda}, \text{ and,} \\ EU^{XP} \geq EU^{UR} &\iff a \leq a_{3-B} \equiv p - \sqrt{\lambda}. \end{aligned}$$

     For the values of $p$ and $\lambda$ in the region we have that:

     $$a_{0-B} > a_{3B} > a_{2B-1}.$$

     Therefore, the demand for the good when $q < \frac{1}{2}$ comprises consumers who buy based on the user reviews and those who do so after asking the expert, and is given by:

     $$ED^{G-2} = (1 - a_{3B}) + \int_{a_{2B-1}}^{a_{3B}} (1 - (p - a))da = \frac{31}{24} - p.$$

  Thus, the expected demand for the region is:

  $$D^G = \frac{1}{2}ED^G + \frac{1}{2}ED^{G-2} = \frac{55}{48} - \frac{p}{2}.$$

  From solving the maximization problem we get the candidate solution $\frac{55}{48}$, which falls outside of the support. Thus, the optimal price is a corner solution: $p^G = 0.6555$. The monopolist serves a demand $D^G = 0.1778$ and obtains profits $\Pi^G = 0.1166$ in this region.

- **When** $p \in \left(0.6555, \frac{3}{4}\right]$

  In this region the expert charges a fee $\lambda = \frac{4p^2}{81}$. There are two sub-cases to consider depending on the size of $q$.

1. When $q \geq \frac{1}{2}$:
   The relevant consumer decisions to calculate the demand for the good are:

$$\begin{aligned} EU^{UR} \geq 0 &\iff a \geq a_0 \equiv p - \frac{3}{4}, \\ EU^{XP} \geq 0 &\iff a \geq a_3 \equiv p - 1 + \sqrt{\lambda}, \text{ and,} \\ EU^{XP} \geq EU^{UR} &\iff a \leq a_4 \equiv p - \frac{1}{2} - \sqrt{\lambda}. \end{aligned}$$

   For the values of $p$ and $\lambda$ in the region we have that:

$$a_{0-B} > a_4 > 0 > a_3.$$

   Therefore, the demand for the good when $q \geq \frac{1}{2}$ comprises consumers who buy based on the user reviews and those who do so after asking the expert, and is given by:

$$ED^{G-2} = (1 - a_4) + \int_0^{a_4} (1 - (p - a))da = \frac{9}{8} + \frac{p(18 - 77p)}{162}.$$

2. When $q < \frac{1}{2}$:
   The relevant consumer decisions to calculate the demand for the good are:

$$\begin{aligned} EU^{UR} \geq 0 &\iff a \geq a_{0-B} \equiv p - \frac{1}{4}, \\ EU^{XP} \geq 0 &\iff a \geq a_{2B-1} \equiv p - \frac{1}{2} + \sqrt{\lambda}, \text{ and,} \\ EU^{XP} \geq EU^{UR} &\iff a \leq a_{3-B} \equiv p - \sqrt{\lambda}. \end{aligned}$$

   For the values of $p$ and $\lambda$ in the region we have that:

$$a_{0-B} > a_{3B} > a_{2B-1}.$$

   Therefore, the demand for the good when $q < \frac{1}{2}$ comprises consumers who buy based on the user reviews and those who do so after asking the expert, and is given by:

$$ED^{G-2} = (1 - a_{3B}) + \int_{a_{2B-1}}^{a_{3B}} (1 - (p - a))da = \frac{763 - 16p(36 + p)}{648}.$$

Thus, the expected demand for the region is:

$$D^G = \frac{1}{2}ED^G + \frac{1}{2}ED^{G-2} = \frac{373 - 9p(14 + 9p)}{324}.$$

From solving the maximization problem we get the candidate solution 0.8245, which falls outside of the support. Thus, the optimal price is a corner solution: $p^G = 0.75$. The monopolist serves a demand $D^G = 0.7189$ and obtains profits $\Pi^G = 0.5392$ in this region.

- **When** $p \in \left(\frac{3}{4}, \frac{9}{11}\right]$

  In this region the expert charges a fee $\lambda = \frac{4p^2}{81}$. There are two sub-cases to consider depending on the size of $q$.

1. When $q \geq \frac{1}{2}$:
   The relevant consumer decisions to calculate the demand for the good are:

$$EU^{UR} \geq 0 \iff a \geq a_0 \equiv p - \frac{3}{4},$$
$$EU^{XP} \geq 0 \iff a \geq a_3 \equiv p - 1 + \sqrt{\lambda}, \text{ and,}$$
$$EU^{XP} \geq EU^{UR} \iff a \leq a_4 \equiv p - \frac{1}{2} - \sqrt{\lambda}.$$

   For the values of $p$ and $\lambda$ in the region we have that:

$$a_{0-B} > a_4 > 0 > a_3.$$

   Therefore, the demand for the good when $q \geq \frac{1}{2}$ comprises consumers who buy based on the user reviews and those who do so after asking the expert, and is given by:

$$ED^{G-2} = (1 - a_4) + \int_0^{a_4} (1 - (p - a))da = \frac{9}{8} + \frac{p(18 - 77p)}{162}.$$

2. When $q < \frac{1}{2}$:
   The relevant consumer decisions to calculate the demand for the good are:

$$EU^{UR} \geq 0 \iff a \geq a_{0-B} \equiv p - \frac{1}{4},$$
$$EU^{XP} \geq 0 \iff a \geq a_{2B-1} \equiv p - \frac{1}{2} + \sqrt{\lambda}, \text{ and,}$$
$$EU^{XP} \geq EU^{UR} \iff a \leq a_{3-B} \equiv p - \sqrt{\lambda}.$$

   For the values of $p$ and $\lambda$ in the region we have that:

$$a_{0-B} > a_{3B} > a_{2B-1}.$$

   Therefore, the demand for the good when $q < \frac{1}{2}$ comprises consumers who buy based on the user reviews and those who do so after asking the expert, and is given by:

$$ED^{G-2} = (1 - a_{3B}) + \int_{a_{2B-1}}^{a_{3B}} (1 - (p - a))da = \frac{763 - 16p(36 + p)}{648}.$$

Thus, the expected demand for the region is:

$$D^G = \frac{1}{2}ED^G + \frac{1}{2}ED^{G-2} = \frac{373 - 9p(14 + 9p)}{324}.$$

From solving the maximization problem we get the candidate solution 0.8245, which falls outside of the support. Thus, the optimal price is a corner solution: $p^G = 0.75$. The monopolist serves a demand $D^G = 0.75$ and obtains profits $\Pi^G = 0.5625$ in this region.

- **When** $p \in \left(\frac{9}{11}, 1\right]$

  In this region the expert charges a fee $\lambda = \frac{1}{36}$. There are two sub-cases to consider depending on the size of $q$.

1. When $q \geq \frac{1}{2}$:
   The relevant consumer decisions to calculate the demand for the good are:

$$
\begin{aligned}
EU^{UR} \geq 0 &\iff a \geq a_0 \equiv p - \frac{3}{4}, \\
EU^{XP} \geq 0 &\iff a \geq a_3 \equiv p - 1 + \sqrt{\lambda}, \text{ and,} \\
EU^{XP} \geq EU^{UR} &\iff a \leq a_4 \equiv p - \frac{1}{2} - \sqrt{\lambda}.
\end{aligned}
$$

   For the values of $p$ and $\lambda$ in the region we have that:

$$a_{0-B} > a_4 > 0 > a_3.$$

   Therefore, the demand for the good when $q \geq \frac{1}{2}$ comprises consumers who buy based on the user reviews and those who do so after asking the expert, and is given by:

$$ED^{G-2} = (1 - a_4) + \int_0^{a_4} (1 - (p - a))da = \frac{11}{9} - \frac{p^2}{2}.$$

2. When $q < \frac{1}{2}$:
   The relevant consumer decisions to calculate the demand for the good are:

$$
\begin{aligned}
EU^{UR} \geq 0 &\iff a \geq a_{0-B} \equiv p - \frac{1}{4}, \\
EU^{XP} \geq 0 &\iff a \geq a_{2B-1} \equiv p - \frac{1}{2} + \sqrt{\lambda}, \text{ and,} \\
EU^{XP} \geq EU^{UR} &\iff a \leq a_{3-B} \equiv p - \sqrt{\lambda}.
\end{aligned}
$$

   For the values of $p$ and $\lambda$ in the region we have that:

$$a_{0-B} > a_{3B} > a_{2B-1}.$$

   Therefore, the demand for the good when $q < \frac{1}{2}$ comprises consumers who buy based on the user reviews and those who do so after asking the expert, and is given by:

$$ED^{G-2} = (1 - a_{3B}) + \int_{a_{2B-1}}^{a_{3B}} (1 - (p - a))da = \frac{31}{24} - p.$$

Thus, the expected demand for the region is:

$$D^G = \frac{1}{2} ED^G + \frac{1}{2} ED^{G-2} = \frac{181 - 36p(2 + p)}{144}.$$

From solving the maximization problem we get a candidate solution which falls outside of the support. Thus, the optimal price is a corner solution: $p^G = 0.8181$. The monopolist obtains profits $\Pi^G = 0.5567$ in this region.

Finally, by comparing the equilibrium profits in all the regions, we can see that the firm obtains the highest profit level setting a price $p^G = 0.75$.

□